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ANALYSIS OF MOMENT ALGORITHMS FOR BLURRED IMAGES

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***Abstract*— with the remarkable growth in image processing, the requirements for dealing out with blurred images is difficulty in a variety of image processing applications. In this paper presents the restoration of blurred images which gets degraded due to diverse atmospheric and environmental conditions, Blur is a key determinant in the sensitivity of image quality, so it is essential to restore the original image.The research outcomes exhibit the major identified bottleneck for restoration is to deal with the blurred images and also a set of attempts have been executed in image restoration using multiple moment algorithms. However the precise results are not been proposed and demonstrated in the comparable researches. Also detail understanding for applications of moment algorithms for image restoration and demonstrating most suitable moment method is current requirements for research. Hence in this work we employ most accepted moment algorithms to exhibit the effect of moments for image restoration and the performance of the moment algorithms such as the Hu, Zernike and Legendre moments is evaluated on image with different blurring lengths. Moreover the effect of moment algorithms is also demonstrated in order to find the optimal setting of orders for image restoration. The final outcome of this work is a stable version of MATLAB based application to visually demonstrate the performance difference of Hu, Zernike and Legendre moments. The relative performance of the application is also been demonstrated with the help of multiple image datasets of biometric identifier such as fingerprint, hand palm and human face.**

***Index Terms*— Image Descriptors, Moment Algorithm, Image blurring, Zernike moment, Legendre Moment, Image Restoration.**

I. INTRODUCTION

Image processing is dynamic research area that has impact in several fields from remote sensing, traffic Surveillance, Biometric authentication system, robotics, to medicine. 3-D scene analysis and reconstruction are only a few objectives to deal with. Since the real sensing systems are sometimes lacking and also the environmental conditions are dynamic over time, the acquired images often. The image are the for the most part frequent component of information representation and transmission due to the robust nature of information storage and the continuous effort to make digital image processing and presentation better. The studies have shown that the images contain information which is redundant and changing a value may cause errors in the calculation for further steps.

In the space of image processing, the restoration of images is the major expanse of research for many decades. Many researchers have proposed various algorithms and techniques for better restoration of images for various applications. However the collection of image is strongly dependent on the imaging agent. The quality of a image possibly will suffer from a variety of impairments, Still the key bottleneck for better restoration of images are the random distortion and blurring caused to the initial images to be provided as input to the recognition system [1] [2]. The distortion and blurriness of the images are not only dependent on the capture agent, but also depends on the environmental and human errors. The causes of blurriness are studies and classified in four major kinds. Firstly, the focal length of the capture devices, Secondly, during the capture of object in a time irrelevant scale needs to be mapped with the capture speed of the agent to avoid the blurriness [3]. Thirdly, sometimes due to environmental and human causes the stabilization of the capture devices may be disturbed causing the blurriness. Fourthly, the most unavoidable situation, where the object is in higher order of colour range but the relevant background of lower order of colour range causing the blurriness. Thus to remove the effect of blurriness of the image, the most appropriate algorithms to be deployed are the momentary calculation algorithms.

In the field of image and computer vision processing, an image moment is defined as a certain particular weighted average (moment) of the image pixels' intensities, or a function of such image moments. These image moments are helpful to depict image after segmentation. The moment found the basic properties of image such area, centric and various information regarding image orientation and Effects of moments in digital image processing for restoration cannot be ignored as supported by related researches. In general moments are the numeric values used to represent the nature of any functions and identify with the significant properties [3] [4]. The following are mostly used moments algorithms are Hu moment, Zernike moment and the well discussed Legendre algorithms.

The Restoration of images which are independent to their size, position, and orientation are main properties, in the past two decades a number of techniques have been developed to extract image's features invariant to translation, scale and rotation of the images and these moment invariants [2] [10] are generally used in so many applications like scene matching, image recognition, image restoration and image classification. As the actual imaging systems and conditions of the images are generally inadequate and blur is typically described by a convolution of an unidentified original image. The typical approach to blur invariants image restoration is initially to remove the blur of the images. Unfortunately, deblurring is extremely hard to perform, to avoid this difficulty; to find out invariants approach many challenges were faced and the most of the deblurring were implemented using geometric, complex moments or central moments and in theses non orthogonal moments the information redundancy is expected for image. Hence information redundancy is a realistic concern, in particularly when images are blurred and for this motivation the orthogonal moments are alternative of the non-orthogonal moments. Most accepted orthogonal moments are Legendre moment and Zernike moment. Yet the application of moments algorithms are not been studied for digital image restoration with the comparative results for blur to restoration algorithm efficiency mapping. Thus in this work we understand the algorithms of moments calculation proposed by Hu, Zernike and Legendre for image restoration and develop a framework for comparing the visual performance of the restoration process by applying the same algorithms.

The rest of the work is structured as follows, in Section II understanding the basic definitions and mathematical background for constructions of the moment algorithms and image blurring, in Section III we define the characteristics of the blur image and the components for Blurred image restoration process, in Section IV, we demonstrate the approach for Blurred Image restoration using moment algorithm, in Section V we discuss the results tested on multiple image datasets and in Section VI we discuss the conclusions and future scope of this work.

II. BASIC DEFINITIONS AND MATHEMATICAL BACKGROUND FOR MOMENT

In the broad-spectrum the relationship between the real image f(x,y), the acquired image g(x,y) and h(x,y) be Point spread function of the imaging system, the definition of image Blurring is represented as Eq.1 , in the convolution[14][15]:

$$g(x,y) = f(x,y) * h(x,y) \tag{1}$$

In the Equation (1) * refers to the linear convolution and this convolution equations is often used conciliation between universality and simplicity, it is universally adequate to represent many realistic circumstances such as motion blur of a flat scene in case of translational motion, out-of-focus blur of a flat scene, media turbulence blur and motion blur of a 3D scene origin in the region of x or y axis by camera rotation and parallel its easiness allows realistic mathematical treatment.

In various situations we do not have idea of the entire original image for the restoration of image which may be ill-posed, in such circumstances; the information of an incomplete but strong depiction of the image is enough. However, such an illustration should be free of the imaging system and should essentially express those features of the original image, which are not affected by the degradations. We are looking for a function I that is invariant to the degradation of Eq-1 i.e. should hold for any admissible h(x,y). Descriptors fulfilling the following clause are called blur invariants [14] [15] or convolution invariants in Equation (2).

$$I(f) = I(f * h) \tag{2}$$

In the field of Image and computer vision processing researches the calculation for image moments or finding the image descriptors are widely accepted. In case of image and vision processing calculating the image moment which is resulting in the image descriptor is performed after the image segmentation. The image properties like area, centroid, pixel values and object orientation in any images, it can be represented using the image moment.

The moment is a certain weighted average of any pixel considering the neighbourhood pixel values and Image moments are classified into three categories as Raw Moments, Central Moments and Scale invariant Moments [3]. In this work, we understand the moments in details:

Two dimensional $(i+j)^{th}$ order moment on image function $f(x,y)$, can be represented as

$$M_{ij} = \int_{-\alpha}^{+\alpha}\int_{-\alpha}^{+\alpha} x^i . y^j . f(x,y) dxdy; i,j = 0,1,2,.... \tag{3}$$

The image function $f(x,y)$ is a denoting a piecewise continuous bound function and all orders of moments exist for moment series $\{M_{ij}\}$ is distinctively defined by f(x,y) and in the same way, f(x,y) is also distinctively defined by the moment series $\{M_{ij}\}$.

$$M_{ij} = \sum_x \sum_y x^i . y^j . f(x,y) \tag{4}$$

The moment in Equation (3) may not be invariant if f(x,y) changes by scaling, translating or rotating. This invariant feature is accomplished by central moments, represented in following equation.

$$\mu_{ij} = \int_{-\alpha}^{+\alpha}\int_{-\alpha}^{+\alpha} (x-\bar{x})^i . (y-\bar{y})^j . f(x,y).dxdy; \bar{x} = \frac{M_{10}}{M_{00}}; \bar{y} = \frac{M_{01}}{M_{00}} \tag{5}$$

Here the $\bar{x}$ and $\bar{y}$ are the basic components of the centroid of the image function f(x,y) is defined on the square [-1,1] X [-1,1] and In case of a digital image, central moment can be represented as the following:

$$\mu_{ij} = \sum_x \sum_y (x-\bar{x})^i.(y-\bar{y})^j.f(x,y) \quad (6)$$

The μ_{ij} for centroid moments can be computed on image function f(x, y) is corresponding to the M_{ij} whose canter is shifted to centroid of the image. Hence, the central moments can be considered as translation invariant.

The moment of order (i + j) where x + y ≥ 2 can be achieved by dividing the central moment with 0th moment and scale invariance is achieved by normalization as following:

$$\eta_{ij} = \frac{\mu_{ij}}{\mu_0^{\gamma}}, \gamma = (i+j+2)/2, i+j = 2,3... \quad (7)$$

Among the orthogonal moments the most recognized moments are the Zernike and Legendre moment. Here we understand Zernike and Legendre Moments in detail.

The 2-D Zernike moments [1] [4] [13] of order i with repetition of j of image function f(r, θ) is defined as

$$Z_{ij} = \frac{i+1}{\pi}\int_0^{2\pi}\int_0^1 V_{ij}^*(r,\theta) f(r,\theta) r dr d\theta, |r| \le 1 \quad (8)$$

where (r,θ) is polar coordinate and V_{ij}^* is complex conjugate and Zernike polynomial $V_{ij}(r,\theta)$ of order i with repetition of j (j=0,±1,±2,...) is defined as

$$V_{ij}^*(r,\theta) = R_{ij}(r) e^{pj\theta}; p = \sqrt{x^2+y^2}, \theta = \arctan(y/x) \quad (9)$$

Where, p is the imaginary unit.

The real valued radial polynomial, $R_{ij}(r)$ is given as follows:

$$R_{ij}(r) = \sum_{k=0}^{(i-|j|)/2} \frac{(-1)^k (i-k)! r^{i-2k}}{k!((i+|j|)/2-k)!((i-|j|)/2-k)!} \quad (10)$$

Where 0≤|j|≤i, and i-|j| is even.

The radial moment of order i with repetition j, represented as

$$D_{ij} = \int_0^{2\pi}\int_0^{\infty} r^i e^{-pj\theta} f(r,\theta) r dr d\theta \quad (11)$$

In Cartesian form, the Eq.11 can be defined as

$$D_{ij} = \int_x\int_y (x-py)^{(i+j)/2}(x+py)^{(i-j)/2} f(x,y) r dx dy \quad (12)$$

Where i-j is an even number and radial moment is

$$Z_{ij} = \frac{i+1}{\pi}\sum_{k=j}^{i} B_{ijk} D_{kj} \quad (13)$$

Where the polynomial coefficient, B_{ijk} is defined as

$$B_{ijk} = \frac{(-1)^{(i-k)/2}((i+k)/2)!}{((i-k)/2))!((k+j)/2)!((k-j)/2)!} \quad (14)$$

The Zernike moments in Equation (8), it is simple to define that when an image undergoes a rotation by an angle α, the transformed image moment function Z_{jk}^R is defined by

$$Z_{jk}^R = Z_{ij} e^{-pjx} \quad (15)$$

The Zernike moment algorithm Equation (15) demonstrates the simple rotational transformation and This property show the way to the end of those Zernike moments magnitude of a rotated image function reside the same to the earlier rotation. Thus |Z|, magnitude of Zernike moment, is defined as a rotation invariant feature of the image.

Legendre Moment for of order (i + j) [3][12] is defined as:

$$\lambda_{ij} = \frac{(2i+1)(2j+1)}{4}\int_{-1}^{+1}\int_{-1}^{+1} P_i(x).P_j(x,y).dxdy \quad (16)$$

Where i, j is ranging from 1 to∞.

Hence the k[th] order Legendre polynomial is written as:

$$P_k(x) = \frac{(2k)!}{2^k (k!)^2} x^k - \frac{(2k-k)!}{2^k !(k-1)!(k-2)!} x^{k-2} + ... \text{ K}^{\text{th}} \text{ Term} \quad (17)$$

Where, D(k) = k/2 or (k-1)/2, is an positive integer.

The recurrence relation for calculating the Legendre polynomial is as following:

$$(k+1)P_{k+1}(x) - (2k+1)xP_k(x) + kP_{k-1}(x) = 0 \quad (18)$$

As disused in the previous section of this work, any image function f (x, y) using Legendre polynomial is defined as following:

$$f(x,y) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \lambda_{ij} P_i(x).P_j(y) \quad (19)$$

Where, λ_{ij} Can be calculated on the closed boundary of the same image.

When the Legendre moment of order (i + j) is less than or equal to L, the image descriptor, then the image function can be represented as following:

$$f(x,y,L) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \lambda_{i-j,j} P_{i-j}(x).P_j(y) \quad (20)$$

When the Legendre moments are calculated within the maximum boundary of the image, then the image function can be represented as following:

$$f(x,y,i_{\max},j_{\max}) = \sum_{i=0}^{i_{\max}}\sum_{j=0}^{j_{\max}} \lambda_{i,j} P_i(x).P_j(y) \quad (21)$$

III. Characteristics of Blurred Image

In the Blurred images, the objects vary in terms of contrast and size. The objects in the image can represent large to small item or the items with detailed visibility. The primary effect of the blurriness on the image is to reduce the contrast and visibility of the images. The reduced visibility images causes less detailed information in the images [10] [11].The objects in the images are generally differentiated by the pixel difference between the object and the background at the object edges.

The blurriness of the image actually reduces the pixel difference at the object edges [11].

The blurriness of the image can be considered in terms of units of lengths. The length of the images denotes the blurriness of the image [Table – I].

TABLE I: BLUR VALUE RANGE

Capture Agent Type	Range of Blur Value (In MM)
Gamma Ray Camera	10 to 2
Ultrasonic Camera	5 to 2.1
Magnetic Resonance Camera	3.4 to 1
Computed Thermo graphy Camera	2 to 1.3
Motion Capture Camera	2.8 to 0.3
Radio Active Camera	0.5 to 0.1

IV. Approach for Blurred Image Restoration using Moment Algorithms

Although Zernike and Legendre moments are useful methods for restoration of blur images, in this section, we discuss the presented approach for restoration using the following mathematical equations to express Zernike moments and Legendre moment. Thus the process of restoring the blurred image using Moment algorithm is presented in this frame work [Fig.1].

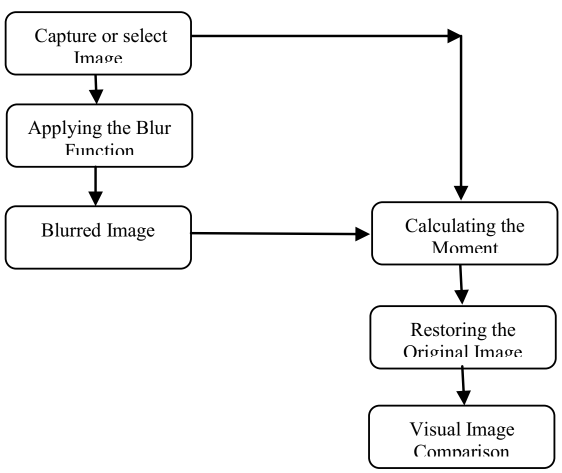

Fig.1. Framework for Blurred Image Restoration

Image restoration procedure using moment algorithms:-

- Capturing image using capture device or select the input image.
- Blur function is applied on the original image.
- Transform original and blurred image into two dimensional, real valued and numeric forms.
- Calculate the image moment using moment algorithms such as Hu, Zernike and Legendre on original image.
- Calculate the image moment using moment algorithms such as Hu, Zernike and Legendre on blur image.
- Restore the blurred image using moment algorithms.
- Visual comparison of original blurred and restored image.

The image blurring is typically illustrated by the convolution in Eq.1 and Point spread function is assumed h(x,y) is a centrally symmetric image function and also the energy preserving image system [16] is represented as

$$h(x,y) = h(-x,-y); \int_{-1}^{1}\int_{-1}^{1} h(x,y)dxdy = 1 \tag{22}$$

The centroid of the blurred image g(x,y) is represents to the centroid of the input image f(x,y) and h(x,y) is point spread function

$$g(\bar{x}) = f(\bar{x}) + h(\bar{x}); g(\bar{y}) = f(\bar{y}) + h(\bar{y}) \tag{23}$$

If h(x,y) is centrally symmetric , then;

$$h(\bar{x}) = h(\bar{y}) = 0; g(\bar{x}) = f(\bar{x}), g(\bar{y}) = f(\bar{y}).$$

Zernike Moment [4] is a most helpful in building rotational moment invariant and property of the blur invariants for blur degradation is examined. Zernike moments of blurred images with respect to original image and PSF can be expressed by following theorem.

The g(x, y) is blurred image is resultant of convolution image f(x, y) with Point Spread Function (PSF) can be represented [4] as follows:

$$Z_{ij}^{g} = \frac{i+1}{\pi}\sum_{k=j}^{i}\sum_{m=0}^{\frac{k+j}{2}}\sum_{n=0}^{\frac{k-j}{2}} \begin{pmatrix} \frac{k+j}{2} \\ m \end{pmatrix}\begin{pmatrix} \frac{k-j}{2} \\ n \end{pmatrix} B_{ijk} D^{h}_{k-m-n,\, j-m+nD^{f}_{m+n,m-n}} \tag{24}$$

Here we imagine that h(x,y) is centrally symmetric point spread function and this system is energy preserving

The two dimensional Legendre Moment [3][12] for the blurred image of g (x, y) can be defined as:

$$L_{i,j}(g) = \int_{-1}^{+1}\int_{-1}^{+1} P_i(x).P_j(y).g(x,y).dxdy \tag{25}$$

With the understanding of blurriness effect on the image, the image pixel will be multiplied by random value generated by the noise function.

$$L_{i,j}(g) = \int_{-1}^{+1}\int_{-1}^{+1} P_i(x).P_j(y).(f*h).dxdy \tag{26}$$

The shift of the pixel values are being calculated in order to find the moment value

$$L_{i,j}(g) = \int_{-1}^{+1}\int_{-1}^{+1} P_i(x).P_j(y).(\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h(p,q)f(x-p,y-q)didj).dxdy \tag{27}$$

Hence the final Legendre moment of the blurred image can be represented as

$$L_{i,j}(g) = \int_{-1}^{+1}\int_{-1}^{+1} h(p,q).(\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} P_i(x+p).P_j(y+q)f(x,y)dxdy)dp.dq \quad (28)$$

Hence Equation (28) defines the calculation of Legendre Moment for blurred image.

V. RESULTS AND DISCUSSIONS

In order to prove the findings and theoretical construction proposed in this work, we provide the MATLAB implementation of this framework to test the visual advantages of available moment algorithms for restoration of blurred images. MATLAB is a highly popular multipurpose numeric programming language for the wide variety of build in library functions ranging from image processing to higher order numeric calculation.

In this section, we have considered Biometric identifiers image dataset of fingerprint, hand palm and human face for restoration using various methods such as Hu, Zernike and Legendre moments.

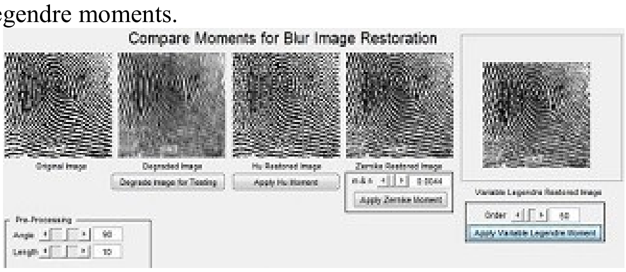

Fig.2. Restoration of fingerprint Image using moments

The input fingerprint image is blurred with length of 10mm and been tested for restoration with Hu, Zernike and Legendre moments of 50 order [Fig.2].

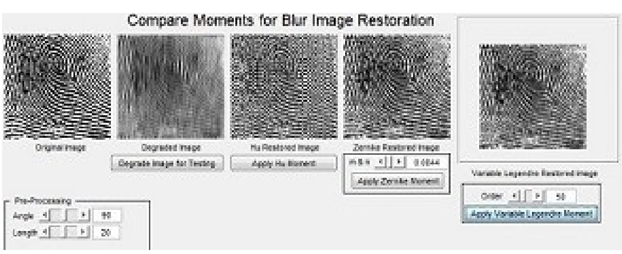

Fig.3. Restoration of fingerprint Image using moments

The input fingerprint image is blurred with length of 20mm and been tested for restoration with Hu, Zernike and Legendre moments of 50 order [Fig.3].

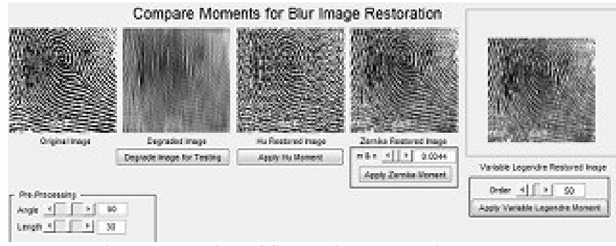

Fig.4. Restoration of fingerprint Image using moments

The input fingerprint image is blurred with length of 30mm and been tested for restoration with Hu, Zernike and Legendre moments of 50 order [Fig.4.].

Fig.5.Restoration of Hand Palm Image using moments

The input hand palm image is blurred with length of 10mm and been tested for restoration with Hu, Zernike and Legendre moments of 50 order [Fig.5].

Fig.6. Restoration of Hand Palm Image using moments

The input hand palm image is blurred with length of 20mm and been tested for restoration with Hu, Zernike and Legendre moments of 50 order [Fig.6].

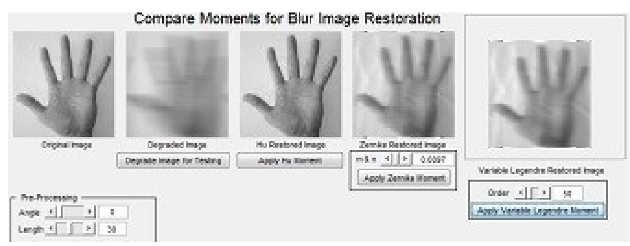

Fig.7. Restoration of Hand Palm Image using moments

The input hand palm image is blurred with length of 30mm and been tested for restoration with Hu, Zernike and Legendre moments of 50 order [Fig.7].

Fig.8. Restoration of Face Image using moments

The input face image is blurred with length of 10mm and been tested for restoration with Hu, Zernike and Legendre moments of 50 order [Fig.8].

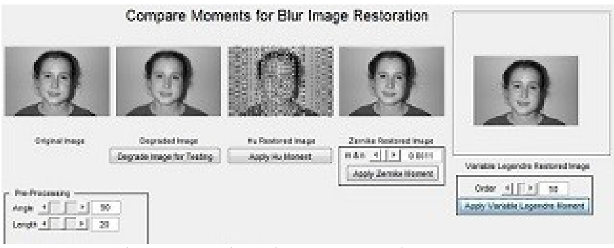

Fig.9.Restoration of Face Image using moments

The input face image is blurred with length of 20mm and been tested for restoration with Hu, Zernike and Legendre moments of 50 order [Fig.9].

Fig.10. Restoration of Face Image using moments

The input face image is blurred with length of 30mm and been tested for restoration with Hu, Zernike and Legendre moments of 50 order [Fig.10].

Henceforth we compare the initial image and restored image generated by the Hu, Zernike and Legendre moments using the following formulation.

The variation between the input image(original) and the output image (restored) with moment algorithms considered as K_1 and the difference between the input image and blurred image is considered as K_2.,Hence the comparative difference between the K_1 and K_2 is considered K, demonstrating the amount of successful restoration for any given image using any given moment algorithm.

$$\left|\det(I_{ori}) - \det(I_{res(Moment)})\right| = K_1 \quad (29)$$

$$\left|\det(I_{ori}) - \det(I_{blur})\right| = K_2 \quad (30)$$

$$|K_1 - K_2| = K, K_1 \to 0, K \to K_2 \quad (31)$$

TABLE II: COMPARATIVE STUDY MOMENT ALGORITHMS BASED ON K VALUE IN EQ. 31.

Input Image	**Blur Length**	**Hu Moment (In %)**	**Zernike Moment (In %)**	**Legendre Moment (In %)**
Fingerprint	10 mm	73	75	81
	20 mm	71	74	77
	30 mm	69	71	73
Hand Palm	10 mm	78	73	69
	20 mm	69	66	63
	30 mm	66	63	59
Human Face	10 mm	37	53	81
	20 mm	41	57	83
	30 mm	53	61	87

The testing results clearly demonstrate the comparative study on biometric identifier such as fingerprint, hand palm and human face for restoration using Hu, Zernike and Legendre moment. For Hand Palm Hue method exhibit better results, Zernike and Legendre show better results for Fingerprint. In the case of human face Legendre moments demonstrates better results.

VI. Conclusion

The considerable amount of analyses has been accomplished with moments such as geometric and orthogonal for restoration of blurred images. The discussion of fundamental mathematical behind those moment algorithms is presented and we have also understood the nature of blurred images. The presented moment algorithms approach builds invariant to blur for restoration of blurred images. Also we demonstrated the fundamental mathematics of moment algorithms such as Hu, Zernike and a Legendre moment on images in order to calculate the image descriptor. The understanding of the difference of lengths for normal and blurred image based on the length for various capture device types also presented. Henceforth, this work proposes a theoretical framework using moments to restore blurred images. The theoretical model is also validated using the application and the results are also been tested. The result of application framework is satisfactory for restoring the blurred images. The application is been tested for biometric identifier images such as Fingerprint, Hand palm and Human face. For majority of the image restoration Legendre moments demonstrate good results with proposed approach.

References

[1]. B. Chen , H. Shu , H. Zhang , G. Coatrieux , L. Luo and J. L. Coatrieux "Combined invariants to similarity transformation and to blur using orthogonal Zernike moments", *IEEE Trans. Image Process.,* vol. 20, no. 2, pp.345 -360 2011.

[2]. J. Flusser , T. Suk and B. Zitová Moments and Moment Invariants in *Pattern Recognition,* 2009 :Wiley

[3]. H. Zhang , H. Shu , G.-N. Han , G. Coatrieux , L. Luo and J. L. Coatrieux "Blurred image recognition by Legendre moment invariants", *IEEE Trans. Image Process.,* vol. 19, no. 3, pp.596 -611 2010

[4]. H. Zhu , M. Liu , H. Ji and Y. Li "Combined invariants to blur and rotation using Zernike moment descriptors", *Pattern Anal. Appl.,* vol. 3, no. 13, pp.309 -319 2010

[5]. Xu, G. S.: 'Sub-pixel edge detection based on curve fitting', *Proc. 2nd Int. Conf. Information and Computing Science, Manchester, UK,* May 2009, pp. 373-375

[6]. Yao, Y., and Ju, H.: 'A sub-pixel edge detection method based on Canny operator', *Proc. 6th Int. Conf. Fuzzy Systems and Knowledge Discovery, Tianjin, China,* August 2009, pp. 97-100.

[7]. Breder, R., Estrela, V., V., and de Assis, J. T.: 'Sub-pixel accuracy edge fitting by means of B-spline', *Proc. IEEE Int. Workshop Multimedia Signal Processing, Rio de Janeiro, Brazil,* October 2009, pp. 1-5

[8]. S. W. Jung, T. H. Kim and S. J. Ko "A novel multiple image deblurring technique using fuzzy projection onto convex sets", *IEEE Signal Process. Lett.,* vol. 16, no. 3, pp.192 -195 2009.

[9]. L. Li and G. Ma "Recognition of degraded traffic sign symbols using PNN and combined blur and affine invariants*", Proc. 4th ICNC, vol. 3,* pp.515 -520 2008.

[10]. Ojansivu and J. Heikkilä"Image registration using blur-invariant phase correlation*", IEEE Signal Process. Lett.,* vol. 14, no. 7, pp.449 -452 2007

[11]. B. Mahdian and S. Saic"Detection of copy-move forgery using a method based on blur moment invariants", *Forensic Sci. Int.,* vol. 171, no. 2/3, pp.180 -189 2007

[12]. Pew-Thian Yap, Raveendran Paramesran," An Efficient Method for the Computation of Legendre Moments", *IEEE Transactions On Pattern Analysis And Machine Intelligence,* Vol. 27, No. 12, December 2005.

[13]. Reza Kasyauqi Sabhara, Chin-Poo Lee, and Kian-Ming Lim "Comparative Study of Hu Moments and Zernike Moments in Object Recognition", *Smart Computing Review,* vol. 3, no. 3, June 2013.

[14]. Jaroslav Kautsky and Jan Flusser "Blur Invariants Constructed From Arbitrary Moments", *IEEE Signal Process,* vol. 20, no. 12, Dec 2011.

[15]. Jan Flusser, Tomas Suk, Jiri Boldys, and Barbara Zitov□ "Projection Operators and Moment Invariants to Image Blurring",*Ieee Transactions On Pattern Analysis And Machine Intelligence,* Vol. 37, No. 4, April 2015

[16]. Tomas Suk, Jan Flusser "Combined blur and affine moment invariants and their use in pattern recognition*", Pattern Recognition 36* (2003) *2895 – 2907,* Elsevier Ltd, 2011.